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NEWLY DEVELOPED STRESS INCREMENT MEASUREMENT TECHNIQUE IN PLASTIC REGION BASED ON STRESS INVERSION THEORY

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Abstract - Newly developed stress increment measurement technique is presented. Based on stress inversion theory proposed by authors, spontaneous stress increment field can be obtained even if the material is under plastic deformation. We present here an application example of this technique by means of the stress increment field measurement at the crack tip of an aluminum plate under the monotonic loading. Strain field are necessary to solve this type of inverse problem, and laser speckle interferometry method which can measure all three axis displacement is used. Boundary traction is also required as boundary conditions, and this is also measured by laser since the material is elastic or traction-free at the boundary in this study. As the results; (1) stress increment field is obtained, (2) stress concentration is confirmed at the crack tip, (3) propagation of plastic region is observed. Since this technique is non-contact, non-destructive, and can be used to any materials, its application field can be almost unlimited.

1. INTRODUCTION

Numerical analysis techniques such as Finite Element Analysis are used in many engineering fields. When elastic-plastic response of a body is to be known, its constitutive relation is necessary. To obtain a satisfactory constitutive relation, we must establish a well-constructed model, and determine some/many parameters to make it fit to the material behavior. This process requires a lot of experiments under different loading conditions since only one stress-strain relation is gathered by each experiment in most cases. If the stress-strain relation can be measured at multiple points for one specimen without any constitutive relation, the amount of information from each experiment increases tremendously. From this point of view, Hori and Kameda proposed field-to-field inversion formulation [1]. The term 'inversion' here means that 'a stress field of a body can be obtained from a strain field and its boundary condition', compared to usual analysis which obtain stress and strain field from its constitutive relation and bondary condition. This inversion scheme can be widely applied to elastic-plastic materials. Kameda and Koyama reported the example applied to Toyoura Sand [2], and Kameda and Nakase observed the fracture of aluminum plate [3]. The proposed inversion method provides valuable information not only for construction of constitutive relation but also for next generation computation with plenty of memory devices. If well-constructed database between strain rate and stress rate in the limit of infinitely small time increment (the expressions “strain increment” and “stress increment” will be used in the following section) is stored in RAM, instead of using explicitly formulated constitutive relation, we can perform numerical computation with this constitutive relation database. It has a potential to increase computation speed dramatically. Kameda and Ozaki have successfully shown the prototype of data-based finite element analysis for J-2 plasticity [4]. This inversion method has a lot of advantages as written above, however, the technique to obtain displacement field data and boundary condition data can be improved for more accurate analysis. In this study, we propose the improved boundary condition acquisition technique and show the results of stress increment measurements for aluminum plates with a hole.

2. FORMULATION

The stress inversion formulation used in this study is briefly explained here. More detailed discussion can be found in [1].

In a two-dimensional state, the self-equilibrating stress components are generated by using Airy's stress function, a, i.e.,

$$\sigma_{11} = a_{,22}, \quad \sigma_{22} = a_{,11}, \quad \sigma_{12} = -a_{,12}. \tag{1}$$

$\sigma_{11}+\sigma_{22}$ can be written as a function of strain $f(\varepsilon)$, with the relation shown in (1), following Poisson's equation is obtained :

$$a_{,11} + a_{,22} = f(\varepsilon). \tag{2}$$

The required boundary conditions to solve (2) can be obtained by the following procedure (for example, see [5]). Here, we define γ_i with traction t_i along the boundary ∂S as

$$\gamma_i = \int_{\partial S} t_i \mathrm{d}\ell. \tag{3}$$

With normal vector n_i and unit tangential vector s_i, γ_1 between given points A and B, for example, becomes

$$\begin{aligned} \gamma_1 &= \int_A^B (n_1\sigma_{11} + n_2\sigma_{12})\mathrm{d}\ell \\ &= \int_A^B (s_2 a_{,22} + s_1 a_{,12})\mathrm{d}\ell = [a_{,2}]_A^B, \end{aligned} \tag{4}$$

where, $\mathrm{d}(.)/\mathrm{d}\ell = s_1(.)_{,1} + s_2(.)_{,2}$ is used. With the same manipulation, we obtain

$$\gamma_2 = -[a_{,1}]_A^B. \tag{5}$$

The above two relations prescribe the Neumann boundary conditions of Airy's stress function as

$$n_1^p a_{,1} + n_2^p a_{,2} = -n_1^p \gamma_2 + n_2^p \gamma_1 \quad \text{along } \partial S, \tag{6}$$

where n^p is a unit normal at the desired position for boundary condition. Since our target is elastic-plastic body, the above discussion is to be rewritten into incremental form.

Next task is to determine the function $f(\varepsilon)$ in (2). Under plane-stress state, the plane displacement increment component $\mathrm{d}u_3$ and the thickness of body h relate to the stress increment to this direction $\mathrm{d}\varepsilon_{33}$ as

$$\mathrm{d}\varepsilon_{33} = \frac{\mathrm{d}u_3}{h}. \tag{7}$$

The stress increments can be written as the sum of its elastic part $\mathrm{d}\varepsilon^e$, and plastic part $\mathrm{d}\varepsilon^p$, i.e.

$$\mathrm{d}\varepsilon_{ij} = \mathrm{d}\varepsilon_{ij}^e + \mathrm{d}\varepsilon_{ij}^p. \tag{8}$$

From the relation between stress increments and strain increments, we obtain

$$\mathrm{d}\sigma_{ii} = \frac{E}{1-2\nu}(\mathrm{d}\varepsilon_{ii} - \mathrm{d}\varepsilon_{ii}^p), \tag{9}$$

where the summation convention is used ($(.)_{ii} = \mathrm{tr}(.)$). Combining incompressibility of plastic deformation,

$$\mathrm{d}\varepsilon_{11}^p + \mathrm{d}\varepsilon_{22}^p + \mathrm{d}\varepsilon_{33}^p = 0, \tag{10}$$

and from the plane-stress state, $\sigma_{33} = 0$, with (9), we obtain $f(\mathrm{d}\varepsilon)$ ($= \mathrm{d}\sigma_{11} + \mathrm{d}\sigma_{22}$) as the following form,

$$f(\mathrm{d}\varepsilon) = \frac{E}{1-2\nu}(\mathrm{d}\varepsilon_{11} + \mathrm{d}\varepsilon_{22} + \mathrm{d}\varepsilon_{33}). \tag{11}$$

The incremental form of (2), (6) and (11) finally prescribe the Airy's stress function boundary value problem as follows:

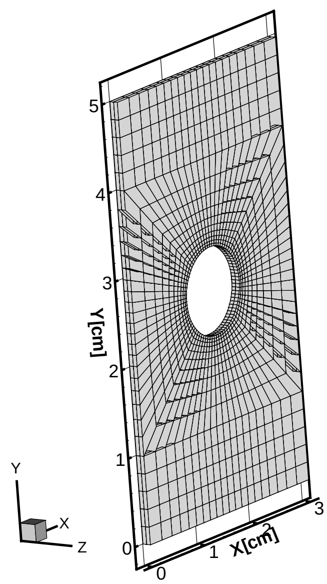
$$\mathrm{d}a_{,11} + \mathrm{d}a_{,22} = f(\mathrm{d}\varepsilon) \quad \textit{on } S \tag{12}$$

$$n_1^p \mathrm{d}a_{,1} + n_2^p \mathrm{d}a_{,2} = -n_1^p \mathrm{d}\gamma_2 + n_2^p \mathrm{d}\gamma_1 \quad \textit{on } \partial S \tag{13}$$

3. NUMERICAL VALIDATION

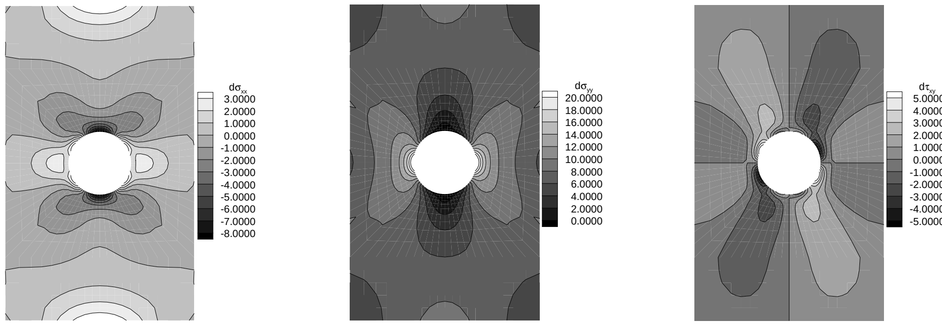
In order to validate the accuracy of this proposed method, a numerical experiment is carried out. A plate with a hole shown in Figure 1 is chosen as the test specimen. It is put under the uniaxial tensile loading. The stress increments obtained by finite element method (FEM) with J-2 plasticity between 0.3 % and 0.31 % tensile strain are used as the reference solution. For inverse analysis, strain increments and boundary traction from FEM are used as the given information, then stress increments are computed by inversion. The results are shown in Figures 2 (a)-(c) for FEM, Figures 3 (a)-(c) for inversion, and the error distributions are shown in Figures 4 (a)-(c), where error is defined as

$$\mathrm{d}\sigma_{err} = \frac{|\mathrm{d}\sigma_{inv} - \mathrm{d}\sigma_{fem}|}{\sqrt{\sum(\mathrm{d}\sigma_{inv} - \mathrm{d}\sigma_{fem})^2}}. \tag{14}$$



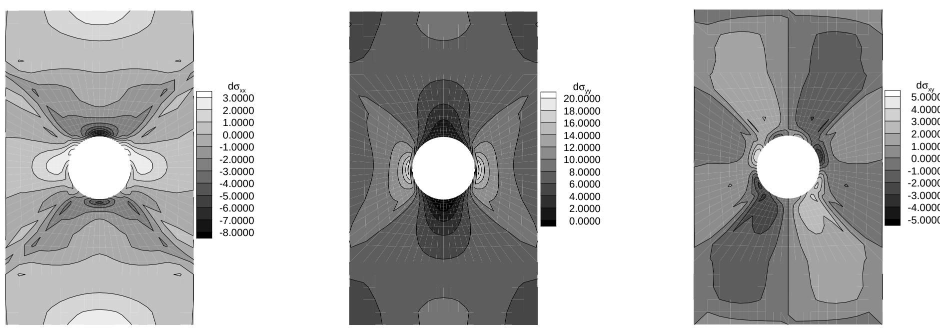
The Number of Nodes	3270
The Number of Elements	2000
Young's Modulus	210×10^4 [kgf/cm^2]
Poisson's Ratio	0.30
Yield Stress	2400 [kgf/cm^2]
Thickness of Plate	0.10[cm]

Figure 1. Specimen for numerical experiment.



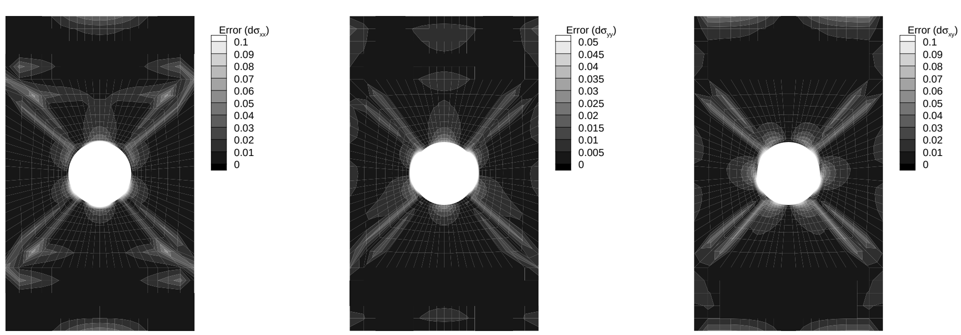
(a) $\mathrm{d}\sigma_{xx}$ (b) $\mathrm{d}\sigma_{yy}$ (c) $\mathrm{d}\sigma_{xy}$

Figure 2. Stress increments from FEM [unit kgf/cm^2].



(a) $\mathrm{d}\sigma_{xx}$ (b) $\mathrm{d}\sigma_{yy}$ (c) $\mathrm{d}\sigma_{xy}$

Figure 3. Stress increments from inversion [unit kgf/cm^2].



(a) Error in $\mathrm{d}\sigma_{xx}$ (b) Error in $\mathrm{d}\sigma_{yy}$ (c) Error in $\mathrm{d}\sigma_{xy}$

Figure 4. Error distributions in stress inversion.

4. EXPERIMENTS AND RESULTS

The proposed inversion method is applied to aluminum plates with a slit for obtaining stress increment fields. The schematic view and dimensions of specimen are shown in Figure 5. While this specimen is pulled by Instron type uniaxial tensile machine (SHIMADZU AGS-H), displacement fields for all directions (X,Y, and Z) are measured by laser speckle interferometry based equipment (ETTEMEYER 3D) with 0.1 μm of sensitivity and 512 $\times$ 512 points/field of resolution. Strain increment fields are calculated from acquired displacement increment fields by their derivatives, and the results are used as given information for the inverse analysis. The final form of Poisson's equation of Airy function is solved by Finite Element Method. Since the problem to be solved is boundary value problem, the accurate measurement of boundary traction is quite important. At the far field from a slit, where it is still elastic, therefore, the stress can be calculated by the accurate measurement of strain. This idea makes us possible to obtain more accurate boundary condition compared to our/others' previous studies which use some assumptions and/or mechanical measurement devices. The global loading behavior between tensile load and crosshead displacement is shown in Figure 6, and the number in the figure shows the moments when stress increment fields are obtained by inversion (I and II). To evaluate the region under plastic deformation, dσ^* is introduced as

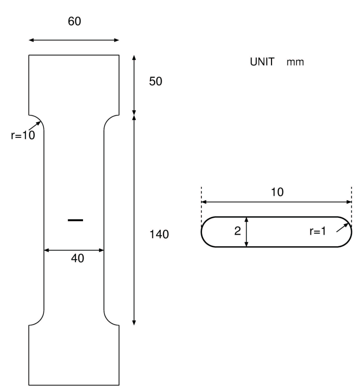
$$\mathrm{d}\sigma^* = \frac{\mathrm{d}\bar{\sigma}^e - \mathrm{d}\bar{\sigma}^{inv}}{\mathrm{d}\bar{\sigma}^e}, \tag{15}$$

where the overbar denotes an effective value by its components, i.e.,

$$\bar{\sigma}(\sigma_{ij}) \quad = \quad \sqrt{\frac{2}{3}(\sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk})(\sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk})}, \tag{16}$$

the superscript e denotes the stress increments calculated from linear elastic relation for given strain increments, and inv denotes the stress increments obtained from inverse analysis. This dσ^* can be used to monitor the growth of plastic region and may be equal to zero at points where locally unloading occurs. The results are shown in Figures 7 and 8 for loading stages I and II, respectively.

Although we have no method to validate the acquired stress increment field at this moment, if there exists a buriable micro-sensor whose effect is negligible, it can be used for calibration. With appropriate calibration, this mesurement technique can be used for stress field observation near the crack, seeking for material parameter without many specimens, and hopefully constructing material behavior database.



Material	thickness [mm]	slit length [mm]
Pure Aluminum A1100	1.0	10.0

Figure 5. Schematic view of specimen and its dimension.

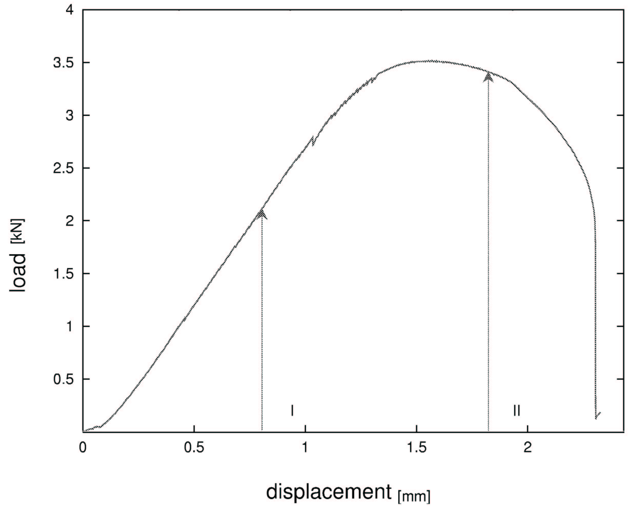
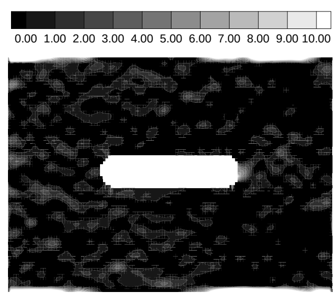
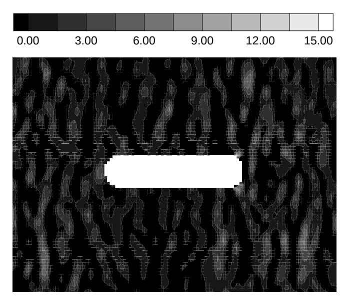


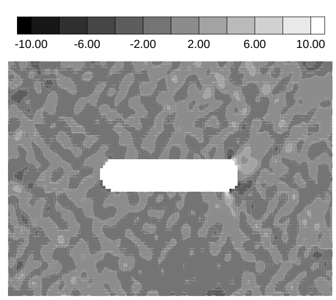
Figure 6. Load-displacement curve.



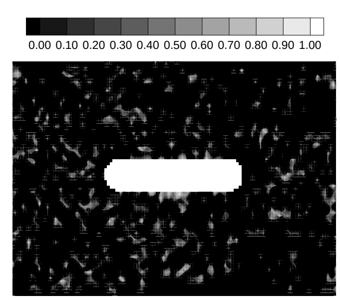
(a) $\mathrm{d}\sigma_{xx}$ (unit MPa)



(b) $\mathrm{d}\sigma_{yy}$ (unit MPa)

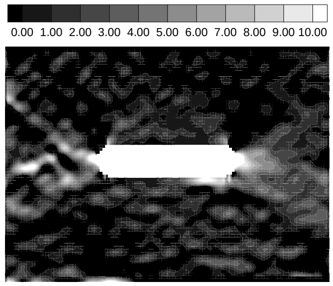


(c) $\mathrm{d}\sigma_{xy}$ (unit MPa)

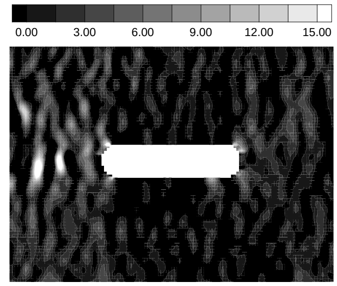


(d) $\mathrm{d}\sigma^{*}$ (normalized by $\mathrm{d}\bar{\sigma}^{e}$)

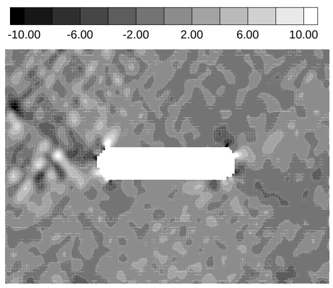
Figure 7. Calculated stress increments at I.



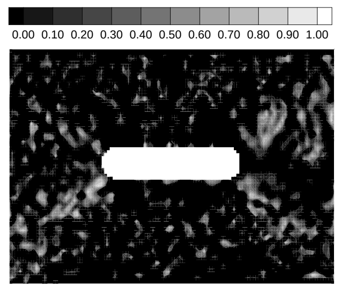
(a) $d\sigma_{xx}$ (unit MPa)



(b) $d\sigma_{yy}$ (unit MPa)



(c) $d\sigma_{xy}$ (unit MPa)



(d) $d\sigma^*$ (normalized by $d\bar{\sigma}^e$)

Figure 8. Calculated stress increments at II.

5. CONCLUSIONS

The numerical experiment shows that the proposed method reproduces the stress increment field from the given strain increment field and the boundary condition, even under the plastic deformed state. The stress field obtained by this inverse technique depends upon not only the acquired strain field and but also the boundary condition. It is important to establish a suitable technique to determine the boundary traction. If it is known that the surrounded region is elastic, the proposed scheme is convenient and reliable since both strain and traction can be measured at once and laser strain analyzer is accurate enough in most purpose. The inversion technique was applied into the real aluminum plate with slit, and the developing stress field around the slit tip region were captured. The calibration of the measurement is an important task for a practical use in future. The stress field (not the stress increment field) measurement which requires continuous measurement has not been achieved yet. In order to complete this, precision slower speed quasi-static loading equipment or higher speed shutter and image processing for laser speckle method is necessary.

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